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THERMAL RADIATION TO A FLAT SURFACE ROTATING ABOUT AN ARBITRARY AXIS IN AN ELLIPTICAL EARTH ORBIT:

APPLICATION TO SPIN-STABILIZED SATELLITES

by Edward I. Powers

Goddard Space Flight Center
Greenbelt, Maryland

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SUMMARY

The derivation of total thermal radiation incident upon a flat plate rotating about an arbitrary axis is presented. The functional relationships between position in an elliptical earth orbit and direct solar radiation, earth-reflected solar radiation (albedo), and earth-emitted radiation (earthshine) are included. The equations have been programmed for the IBM 7090 digital computer, resulting in solutions which relate the incident radiation to spin axis orientation and orbital position. Several representative orbits for a typical geometrical configuration were analyzed and are presented as examples.

CONTENTS

Summary i

INTRODUCTION. 1

COORDINATE SYSTEMS AND VECTOR REPRESENTATION 3

PLATE ROTATING ABOUT THE SPIN AXIS 5

DETERMINATION OF EARTHSHINE 7

DETERMINATION OF ALBEDO 9

CALCULATION OF DIRECT SOLAR FLUX. 10

APPLICATION. 11

Acknowledgments 12

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INTRODUCTION

The satisfactory operation of an artificial satellite depends upon maintaining the payload temperature within prescribed limits. For example, the standard batteries employed in present-day spacecraft generally restrict the temperature limits to 0° and 40°C. Often experiments located within the satellite structure further restrict this variation.

The temperature level of a satellite may be determined by solving the instantaneous energy balance

$$P + S \alpha_s A_p + q_{alb} + q_{es} = \sigma \epsilon A_s \bar{T}^4 + WC_p \frac{dT}{dt} ,$$

where

P = internal power dissipation,

S = solar constant,

α_s = solar absorptance,

A_p = instantaneous projected area for sunlight,

q_{alb} = reflected solar radiation (albedo),

q_{es} = earth-emitted radiation (earthshine),

σ = Stefan-Boltzmann constant,

ϵ = infrared emittance,

A_s = total surface area,

$\overline{T}^4$ = mean fourth power, surface temperature,

WC_p = heat capacity of the satellite,

$\frac{dT}{dt}$ = time rate of change of satellite temperature.

If the average orbital temperature is being computed, the last term is dropped. In this case all heat input terms represent integrated orbital values.

It should be noted that the above equation, as a representation of the entire satellite, is greatly oversimplified. The values for ϵ and α_s generally vary over the surface and great fluctuations in skin temperature may exist. In practice the thermal analysis consists of the development of a thermal model which represents a fine mesh of interconnected isothermal nodes. The appropriate relationship between nodes in regard to thermal conduction and radiation interchange must be established.

The energy balance for each node of a thermal model may be written

$$P_n + S\alpha_{s_n} A_{p_n} + q_{alb_n} + q_{es_n} + \sum_m C_{nm}\left(T_m - T_n\right) + \sigma \sum_m E_{nm} F_{nm}\left(T_m^4 - T_n^4\right) = \sigma \epsilon A_{s_n} T_n^4 + \left(WC_p\right)_n \frac{dT_n}{dt} ,$$

where

C_{nm} = conductance between nodes n and m,

E_{nm} = effective emissivity between nodes n and m,

F_{nm} = shape factor-area product between nodes n and m.

Since the major interest at present is to determine the satellite temperature level and not the gradients within, a discussion of the terms in the first equation is in order.

For most passive controlled satellites P is relatively small compared with the total radiation input and does not have a significant effect on the satellite mean temperature. The magnitude of this effect, however, depends upon the ϵ of the surface (e.g. a surface with a low absolute ϵ may raise the internal temperature significantly because the skin has a limited capacity for re-radiation).

The remaining three heat sources, direct solar heating, earth-reflected solar heating (albedo), and earth-emitted radiation (earthshine), represent the significant inputs to the satellite. It is apparent that an adequate thermal design is predicated upon a reasonably accurate knowledge of these thermal radiation inputs. The major source of energy—direct solar radiation—is fortunately the most accurately obtained. Since the sun's rays impinging upon the satellite are virtually parallel, the problem is simply one of determining the instantaneous orientation of each external face with respect to the solar vector.

The calculation of earthshine is considerably less precise, requiring fundamental assumptions to reduce the complexity of computation. These include the assumptions that the earth is a diffusely emitting blackbody and that the surface temperature is uniform at 450°R. This permits direct calculation of energy input from all visible positions on the earth. Since all locations supply varying inputs, an integral equation must be solved to obtain the total incident energy.

The albedo determination involves a similar integration. The earth in this case is assumed to be a diffusely reflecting sphere. In addition, the source intensity is a function of the satellite location relative to the sunlit portion of the earth.

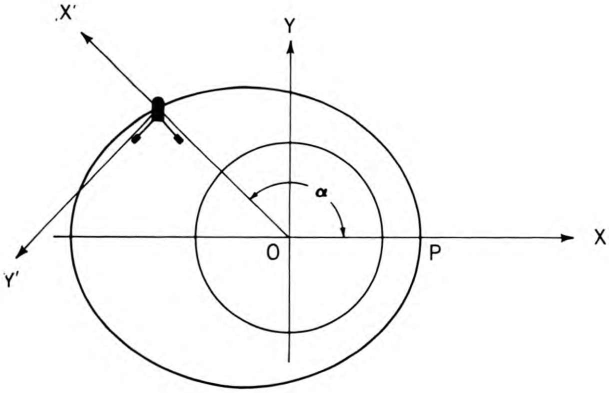
For a nonrotating satellite whose orientation is "fixed in space" the calculation of the thermal radiation fluxes is performed at any interval during the orbit. At each orbital position a particular flat surface may or may not "see" the entire part of the earth's cap visible from the satellite. In the latter case integration for albedo and earthshine must exclude the shaded portion of the cap. Spin-stabilized satellites which rotate uniformly about the spin axis greatly complicate the analysis. The rotation around an arbitrary axis means in general that the heat fluxes vary, since the visible portion of the earth is a function of this rotation.

The analysis presented herein is based upon the energy impinging upon a flat surface whose orientation is defined in vector notation (by the normal vector), and which is rotating about an arbitrary axis. This has a much broader application than may be realized at first. Although the obvious application is for spin-stabilized satellites, the results apply to any body of revolution. Here the interest lies in the variation of flux about the axis rather than an average value per spin. Thus, with the orientation of a single flat plate, the impinging fluxes on cylinders and cones are obtained. A sphere or a shape with a variable surface curvature along its axis requires several or many such plates, depending upon the precision required. It can be seen that the thermal radiation to the entire satellite surface may be found by simply considering a handful of appropriately oriented flat plates.

The purpose of this paper is to present, in general form, the derivation of the governing equations for the radiation energy sources as stated. The equations refer specifically to a flat surface rotating about an axis whose orientation is arbitrary. The dependence upon orbital position is included; the results of the numerical integration of these equations, encompassing a suitable range of applicable orbits, is presented.

COORDINATE SYSTEMS AND VECTOR REPRESENTATION

The primary coordinate system is fixed with respect to the orbit (Figure 1). The XY plane lies in the plane of the orbit with the X axis coincident with the line connecting the center of the earth (the focus of the ellipse) and perigee.



LEGEND
O – EARTH CENTER
P – PERIGEE

Figure 1—Coordinate systems.

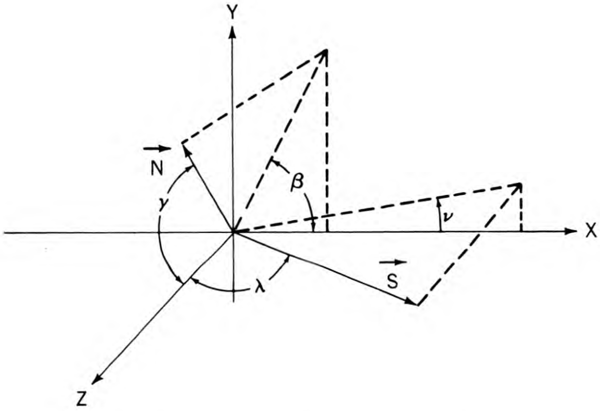


Figure 2—Orientation of normal and solar vectors.

As the satellite traverses the orbit, the instantaneous location is defined by the angle α. The altitude, therefore, may be determined at any instant by

$$A(\alpha) = \frac{(P + R_e)(1 + e)}{1 + e \cos\alpha} - R_e \tag{1}$$

where

$A(\alpha)$ = altitude,

P = altitude at perigee,

R_e = radius of the earth,

e = eccentricity of the orbit.

The orientation of the unit vectors (Figure 2) representing the normals to the flat plates, **N**, and the solar vector, S, are specified by the following angles:

β = angle between the projection of **N** on the XY plane and the X axis,

γ = angle between **N** and the Z axis,

ν = angle between the projection of S on the XY plane and the X axis,

λ = angle between S and the Z axis.

The vectorial representations in the fixed coordinate system are thus

$$\mathbf{N} = \cos\beta \sin\gamma \,\mathbf{i} + \sin\beta \sin\gamma \,\mathbf{j} + \cos\gamma \,\mathbf{k} \tag{2}$$

and

$$\mathbf{S} = \cos\nu \sin\lambda \,\mathbf{i} + \sin\nu \sin\lambda \,\mathbf{j} + \cos\lambda \,\mathbf{k} \ . \tag{3}$$

It is convenient to introduce an instantaneous coordinate system (X′ Y′ Z′) whose origin is fixed in the satellite (Figure 1). The X′Y′ plane lies in the orbital plane with the X′ axis coincident with the line from the earth's center to the satellite. Z′ and Z have the same orientation.

The vectors **N** and S in the primed system are

$$\mathbf{N} = \cos(\beta - \alpha) \sin\gamma \,\mathbf{i}' + \sin(\beta - \alpha) \sin\gamma \,\mathbf{j}' + \cos\gamma \,\mathbf{k}' \ , \tag{4}$$

$$\mathbf{S} = \cos(\nu - \alpha) \sin\lambda \,\mathbf{i}' + \sin(\nu - \alpha) \sin\lambda \,\mathbf{j}' + \cos\lambda \,\mathbf{k}' \ . \tag{5}$$

In a similar manner the expression for the spin axis vector **A** may be shown to be

$$\mathbf{A} = \cos(\delta - \alpha)\sin\mu\ \mathbf{i}' + \sin(\delta - \alpha)\sin\mu\ \mathbf{j}' + \cos\mu\ \mathbf{k}' \tag{6}$$

where

δ = angle between the projection of **A** on the XY plane and the X axis,

μ = angle between **A** and the Z axis.

PLATE ROTATING ABOUT THE SPIN AXIS

For spin-stabilized satellites the normal vectors **N**, which represent the orientation of the exterior surfaces, rotate about the spin axis. In determining both instantaneous and average heat fluxes, the orientation of **N** as a function of this rotation with respect to the primed coordinate system (X′ Y′ Z′) must be known. This is accomplished by defining a third coordinate system X″ Y″ Z″ (Figure 3). The X″ axis is coincident with the spin axis **A**. Y″ is defined so that it lies in the plane formed by X″ and an arbitrary fixed position of the normal vector $\mathbf{N}_o$.*

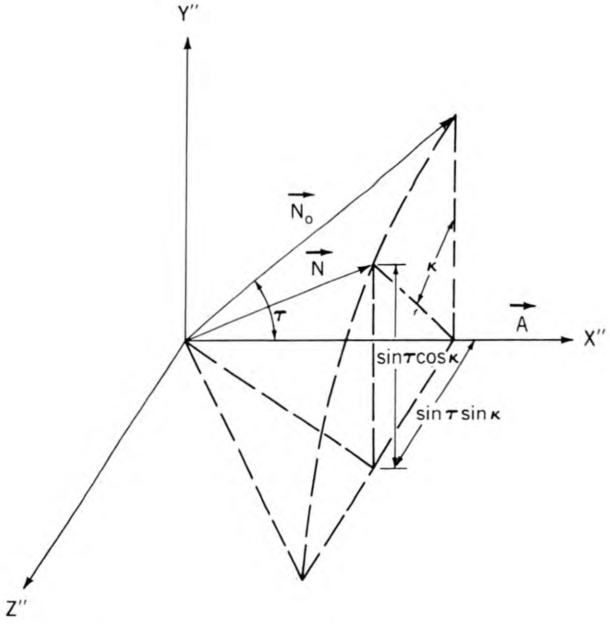


Figure 3—Double-primed coordinate system.

In Figure 3, the following terms may be evaluated:

$$\cos\tau = \cos(\beta - \alpha)\sin\gamma\cos(\delta - \alpha)\sin\mu + \sin(\beta - \alpha)\sin\gamma\sin(\delta - \alpha)\sin\mu + \cos\gamma\cos\mu\ , \tag{7}$$

$$\sin\tau = \sqrt{1 - \cos^2\tau}\ ,$$

$$\mathbf{N} = \cos\tau\ \mathbf{i}'' + \sin\tau\cos\kappa\ \mathbf{j}'' + \sin\tau\sin\kappa\ \mathbf{k}'' \tag{8}$$

It can be seen that the angle between the spin axis and $\mathbf{N}_o$ can be computed only in the first and second quadrants of the X″Y″ plane. This has little effect since κ varies from 0 to 2π (a rotation). [If the vector $\mathbf{N}_o$ lies in the third quadrant (of the X″Y″ plane) the initial position for rotation remains in the second quadrant.] The objective now is to relate the double-primed unit vectors to the primed vectors in order to determine **N** in the primed system.

---

*Since a complete rotation occurs, the orientation of N_o merely indicates the starting point.

The projections of $\mathbf{j}''$ and $\mathbf{k}''$ in the primed coordinate system are found in the following manner: The value for the unit vector $\mathbf{k}''$ is

$$\mathbf{k}'' = \frac{\mathbf{A} \times \mathbf{N}_o}{\left| \mathbf{A} \times \mathbf{N}_o \right|} = \frac{g\mathbf{i}' + h\mathbf{j}' + l\mathbf{k}'}{\sqrt{g^2 + h^2 + l^2}} , \tag{9}$$

where

$$g = \sin(\delta - \alpha) \sin\mu \cos\gamma - \cos\mu \sin(\beta - \alpha) \sin\gamma ,$$

$$h = \cos\mu \cos(\beta - \alpha) \sin\gamma - \cos(\delta - \alpha) \sin\mu \cos\gamma ,$$

$$\begin{aligned} l = {} & \cos(\delta - \alpha) \sin\mu \sin(\beta - \alpha) \sin\gamma \\ & - \sin(\delta - \alpha) \sin\mu \cos(\beta - \alpha) \sin\gamma \end{aligned}$$

Similarly

$$\mathbf{j}'' = \frac{\left(\mathbf{A} \times \mathbf{N}_o\right) \times \mathbf{A}}{\left| \left(\mathbf{A} \times \mathbf{N}_o\right) \times \mathbf{A} \right|} = \frac{m\mathbf{i}' + n\mathbf{j}' + o\mathbf{k}'}{\sqrt{m^2 + n^2 + o^2}} \tag{10}$$

where

$$m = h \cos\mu - l \sin(\delta - \alpha) \sin\mu ,$$

$$n = l \cos(\delta - \alpha) \sin\mu - g \cos\mu ,$$

$$o = g \sin(\delta - \alpha) \sin\mu - h \cos(\delta - \alpha) \sin\mu .$$

$\mathbf{N}$ may now be determined in the primed coordinate system ($X'\,Y'\,Z'$). By rewriting the double-primed unit vectors,

$$\left.\begin{aligned} \mathbf{i}'' &= p\mathbf{i}' + q\mathbf{j}' + r\mathbf{k}' \\ \mathbf{j}'' &= s\mathbf{i}' + t\mathbf{j}' + u\mathbf{k}' \\ \mathbf{k}'' &= v\mathbf{i}' + w\mathbf{j}' + x\mathbf{k}' \end{aligned}\right\} , \tag{11}$$

where

$$p = \cos(\delta - \alpha) \sin\mu ,$$

$$q = \sin(\delta - \alpha) \sin\mu ,$$

$$r = \cos\mu ,$$

$$s = \frac{m}{\sqrt{m^2 + n^2 + o^2}} ,$$

$$t = \frac{n}{\sqrt{m^2 + n^2 + o^2}} ,$$

$$u = \frac{o}{\sqrt{m^2 + n^2 + o^2}} ,$$

$$v = \frac{g}{\sqrt{g^2 + h^2 + l^2}} ,$$

$$w = \frac{h}{\sqrt{g^2 + h^2 + l^2}} ,$$

$$x = \frac{l}{\sqrt{g^2 + h^2 + l^2}} .$$

By substituting for **i**", **j**", and **k**" in Equation 8

$$\mathbf{N} = a\mathbf{i}' + b\mathbf{j}' + c\mathbf{k}' , \tag{12}$$

where

$$a = p \cos\tau + s \sin\tau \cos\kappa + v \sin\tau \sin\kappa ,$$

$$b = q \cos\tau + t \sin\tau \cos\kappa + w \sin\tau \sin\kappa ,$$

$$c = r \cos\tau + u \sin\tau \cos\kappa + x \sin\tau \sin\kappa ,$$

where κ is the angle of rotation. **N** may now be evaluated at any position during rotation throughout the orbit in the primed system $(X' Y' Z')$.

DETERMINATION OF EARTHSHINE

The general expression for the radiation intensity dq, from a diffusely emitting source dA_e, impinging upon a unit area, is as follows:

$$dq = \frac{I \cos\omega \cos\eta \, dA_e}{D^2} , \tag{13}$$

where

I = source intensity,

ω = angle between the line connecting the unit area with dA_e and the normal to dA_e $(\mathbf{N}_e)$,

η = angle between the line connecting dA_e with the unit area and the normal to the unit area,

D = distance between the unit area and dA_e.

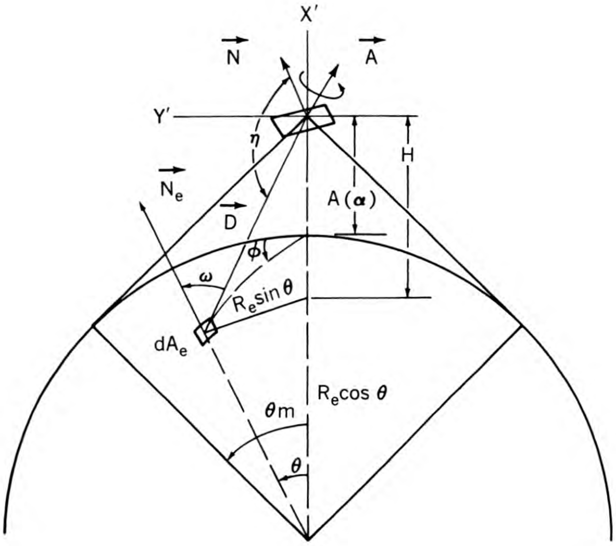


Figure 4—Earth's cap visible from satellite ($\phi = 0$ when coincident with Y').

With the application of this equation to a flat plate of unit area at an altitude $A(\alpha)$, the radiation intensity impinging upon the plate is

$$q_E = \frac{\sigma T_e^4}{\pi} \int_0^{\theta_m} \int_0^{2\pi} \frac{\cos\omega \cos\eta \, dA_e}{D^2} \tag{14}$$

where

σ = Stefan-Boltzmann constant,

T_e = mean black body temperature of the earth's surface,

σT_e^4 = the heat flux leaving the source dA_e.

From the geometry of Figure 4 (which is similar to a sketch used by Katz*) and the vector notation presented, the terms in the equation may be defined more specifically:

$$\mathbf{D} = -H\mathbf{i}' + R_e \sin\theta \cos\phi \, \mathbf{j}' + R_e \sin\theta \sin\phi \, \mathbf{k}' ,$$

$$= -\left[A(\alpha) + R_e (1 - \cos\theta)\right] \mathbf{i}' + R_e \sin\theta \cos\phi \, \mathbf{j}' + R_e \sin\theta \sin\phi \, \mathbf{k}' ,$$

$$D^2 = \left[A(\alpha) + R_e(1 - \cos\theta)\right]^2 + R_e^2 \sin^2\theta ,$$

$$\omega = \theta + \tan^{-1}\left[\frac{R_e \sin\theta}{A(\alpha) + R_e (1 - \cos\theta)}\right] ,$$

$$\cos\eta = \frac{\mathbf{D} \cdot \mathbf{N}}{|\mathbf{D}|} ,$$

$$= \left\{-a\left[A(\alpha) + R_e (1 - \cos\theta)\right] + b R_e \sin\theta \cos\phi + c R_e \sin\theta \sin\phi\right\} \cdot$$

$$\cdot \frac{1}{\sqrt{\left[A(\alpha) + R_e (1 - \cos\theta)\right]^2 + \left(R_e^2 \sin^2\theta\right)}} ,$$

$$\theta_m = \cos^{-1}\left[\frac{R_e}{A(\alpha) + R_e}\right] .$$

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*Katz, A. J., "Determination of Thermal Radiation Incident upon the Surfaces of an Earth Satellite in an Elliptical Orbit," Grumman Aircraft Engineering Corp., Rept. XP 12.20, May 1960.

Equation 14 may now be written in the form

$$q_E = \frac{\sigma T_e^4}{\pi} \int_0^{\cos^{-1}\left[\frac{R_e}{A(\alpha) + R_e}\right]} \int_0^{2\pi} \cos\left\{\theta + \tan^{-1}\left[\frac{R_e \sin\theta}{A(\alpha) + R_e\,(1-\cos\theta)}\right]\right\} \cdot$$

$$\cdot \left\{a\left[-A(\alpha) + R_e\,(1-\cos\theta)\right] + b\,R_e \sin\theta \cos\phi + c\,R_e \sin\theta \sin\phi\right\} \cdot$$

$$\cdot \left\{\frac{1}{\left[A(\alpha) + R_e\,(1-\cos\theta)\right]^2 + R_e^2 \sin^2\theta}\right\}^{3/2} R_e^2 \sin\theta \, d\phi \, d\theta \ . \qquad (15)$$

The limits of integration are such that the part of the earth cap visible from the instantaneous location of the satellite is included. For most practical cases, at least a portion of the cap is not visible from a plate because of its orientation. Attempts to define the appropriate integration limits for such cases are extremely laborious. Since the equations cannot be solved without the aid of a high speed computer, an alternative approach is employed.* The numerical integration includes the entire visible part of the earth cap as stated, but the contributions of the elemental area that the plate does not see are deleted if the local value of $\cos\eta$ is negative.

Equation 15 represents the flux for the instantaneous orientation of the plate. Interest also lies in the determination of the flux for a complete rotation of the plate about the satellite spin axis. The average value for q_E is therefore

$$q_{E_{av.}} = \frac{1}{2\pi} \int_0^{2\pi} q_E \, d\kappa \qquad (16)$$

where κ is the rotational angle.

DETERMINATION OF ALBEDO

The calculation for the albedo input to an orbiting plate is similar to that for the earthshine but involves additional basic assumptions. The reflectance of the earth depends on what is the visible surface—land, water, snow, cloud cover, etc. Until precise data is available which indicates the functional relationship between the albedo and the visible surface, an approximate mean value of 35 percent appears satisfactory.

The reflected radiation is assumed to be diffuse. The local earth-reflected intensity varies with the cosine of the angle between the solar vector and the local normal. Since the input to the plate is a function of the source intensity of the visible elemental areas, the reflected radiation can be expressed by

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*Katz, A. J., op. cit.

$$q_A = \frac{S\ alb}{\pi} \int_0^{\cos^{-1}\left[\frac{R_e}{A(\alpha) + R_e}\right]} \int_0^{2\pi} \cos\left\{\theta + \tan^{-1}\left[\frac{R_e \sin\theta}{A(\alpha) + R_e (1 - \cos\theta)}\right]\right\} \cdot$$

$$\cdot \left\{a \left[- A(\alpha) + R_e (1 - \cos\theta)\right] + b R_e \sin\theta \cos\phi + c R_e \sin\theta \sin\phi\right\} \cdot$$

$$\cdot \cos\psi \cdot \left\{\frac{1}{\left[A(\alpha) + R_e (1 - \cos\theta)\right]^2 + R_e^2 \sin^2\theta}\right\}^{3/2} R_e^2 \sin\theta \, d\phi \, d\theta , \qquad (17)$$

where

S = solar constant (440 BTU/hr/ft^2)

alb = albedo factor (0.35)

ψ = angle between the solar vector $\mathbf{S}$ and the normal to dA_e, $\mathbf{N}_e$,

Cos ψ may be obtained from:

$$\cos\psi = \mathbf{S} \cdot \mathbf{N}_e$$

$$= \cos(\nu - \alpha) \cos\theta \sin\lambda + \sin(\nu - \alpha) \sin\theta \cos\phi \sin\lambda + \cos\lambda \sin\theta \sin\phi . \qquad (18)$$

The area of the earth that contributes to the albedo input is the surface common to the sunlit part of the earth and the part of the cap visible from the plate. The remaining area visible from the plate but receiving no sunlight contributes nothing to the heat input. This is accounted for in the numerical integration by deleting inputs when $\cos\psi$ is negative.

The average albedo flux received by the plate per rotation about the spin axis is

$$q_{A_{av.}} = \frac{1}{2\pi} \int_0^{2\pi} q_A \, d\kappa . \qquad (19)$$

CALCULATION OF DIRECT SOLAR FLUX

The calculation of direct sunlight involves the determination of the instantaneous projected area of the plate with respect to the solar vector:

$$q_{SR} = S (\mathbf{S} \cdot \mathbf{N})$$

$$= S \left[a \cos(\nu - \alpha) \sin\lambda + b \sin(\nu - \alpha) \sin\lambda + c \cos\lambda\right] , \qquad (20)$$

where S is the solar constant.

Because of its orientation the plate may or may not be facing the sun during a rotation. Negative values of $\mathbf{S} \cdot \mathbf{N}$ indicate that it is not facing the sun.

The average flux per spin is

$$q_{SR_{av.}} = \frac{1}{2\pi} \int_0^{2\pi} q_{SR} \, d\kappa \quad . \tag{21}$$

Because the orientation of the plate is fixed in space, the heat flux is constant throughout the sunlit portion of the orbit. To determine whether or not the satellite is within the earth's shadow at any orbital position requires two simple checks.* If both of the following expressions are satisfied, the satellite received no direct input from the sun:

$$\cos\Omega < 0 \quad ,$$

$$\left[R_e + A(\alpha)\right] \cdot \left|\sin\Omega\right| < R_e \quad ,$$

where Ω is the angle between the solar vector $\mathbf{S}$ and X',

$$\cos\,\Omega = \mathbf{S} \cdot \mathbf{i}' \quad ,$$

$$\cos\Omega = \cos\,(\nu - \alpha)\,\sin\lambda \quad .$$

APPLICATION

Hypothetical geometric configurations (Figure 5) have been chosen to illustrate the use of the equations. The external surfaces are represented by the indicated normal vectors.

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*Katz, A. J., op. cit.

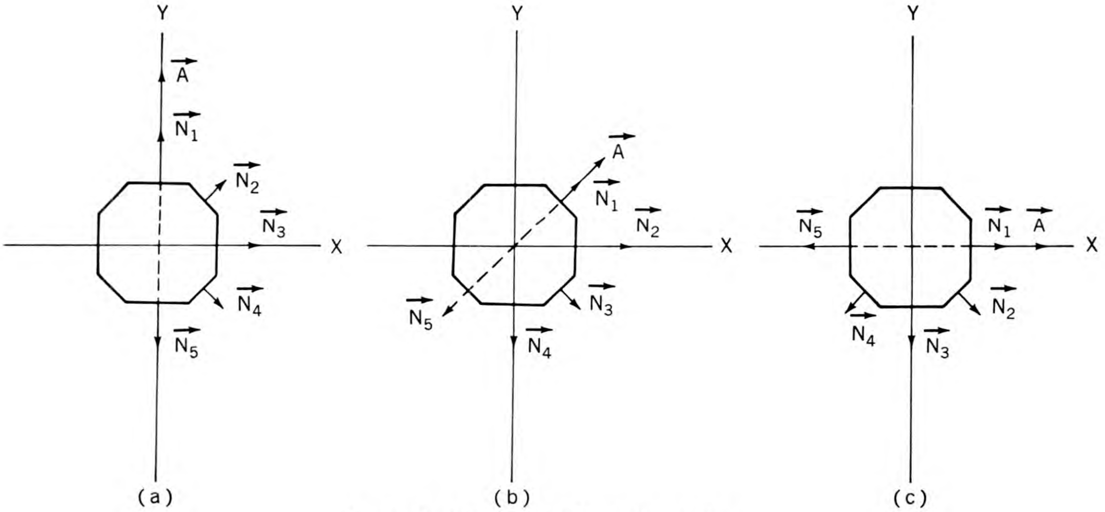

Figure 5—Geometrical configurations.

The three sketches represent three spin axis orientations.

Circular orbits of 150, 550, 1050, 2000, and 5000 statute miles, in which the solar vector lies in the plane of the orbit (minimum sunlight), were considered. Figures 6-35 indicate the orbital variation of earthshine and albedo.* These heat fluxes represent mean integrated values per rotation at the instantaneous orbital position. The direct solar flux is constant in sunlight for a specific orientation. Appropriate values are presented in Table 1.

ACKNOWLEDGMENTS

The author wishes to express gratitude to Mr. Frank Hutchinson and Mr. Henry Hartley for the IBM 7090 program and the data presentation, respectively.

Table 1

Mean Solar Heat Flux per Rotation*

δ (degrees)	β (degrees)	$Q_{SR_{av.}}$ (BTU/hr/ft^2)
90	90	0.0
90	45	99.0
90	0	140.0
90	315	99.0
90	270	0.0
45	45	311.0
45	0	220.0
45	315	99.0
45	270	0.0
45	225	0.0
0	0	440.0
0	315	311.0
0	270	0.0
0	225	0.0
0	180	0.0

*The data presented represent the instantaneous daylight heat flux impinging upon the rotating faces of the configuration in Figure 5. The sun in all cases is parallel to the X axis; μ, γ, and λ are 90 degrees.

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*The mathematical model assumed the earth cap to be comprised of 512 elemental areas. The curves are based primarily on calculations made at 22.5 degree increments throughout the orbit. Because of this, the peaks in some of the albedo curves were estimated. It should be noted that, because of the configurations chosen, the orbital heat fluxes in several cases are identical except for angular displacement.

(Manuscript received June 26, 1963)

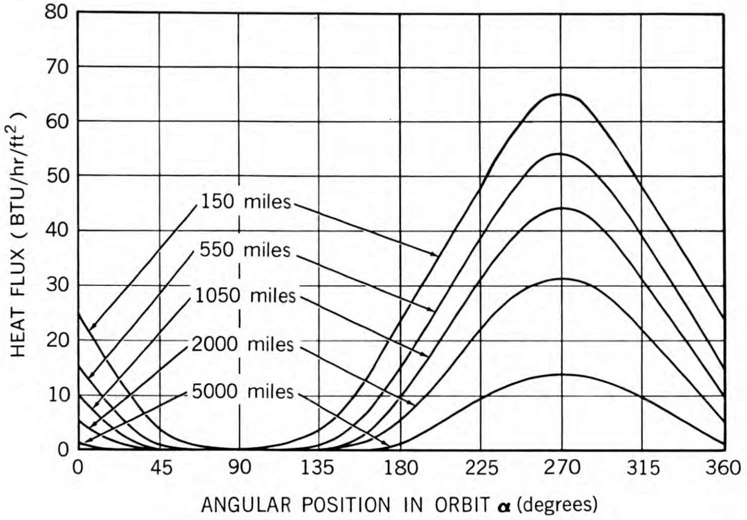

Figure 6—Earth-emitted radiation for circular orbits, δ = 90 degrees, β = 90 degrees.

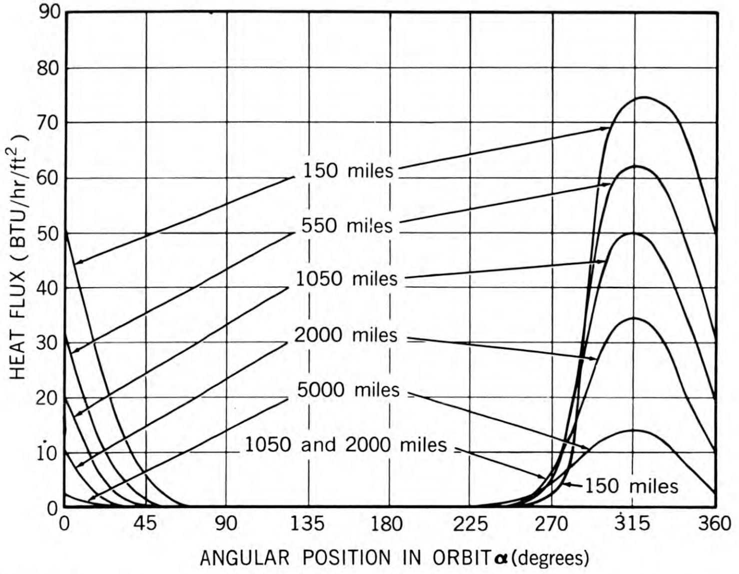

Figure 7—Albedo for minimum sunlit circular orbits, δ = 90 degrees, β = 90 degrees.

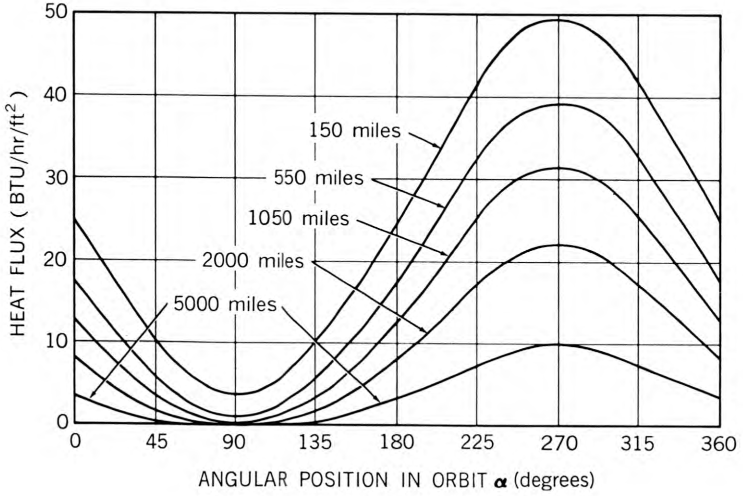

Figure 8—Earth-emitted radiation for circular orbits, δ = 90 degrees, β = 45 degrees.

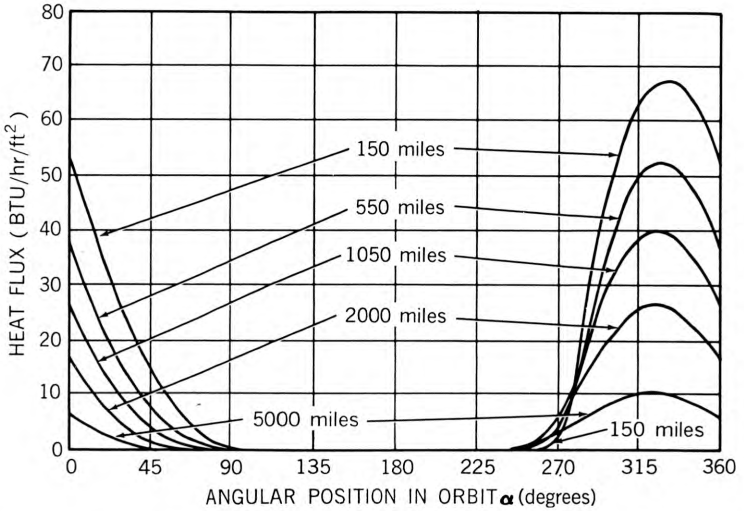

Figure 9—Albedo for minimum sunlit circular orbits, δ = 90 degrees, β = 45 degrees.

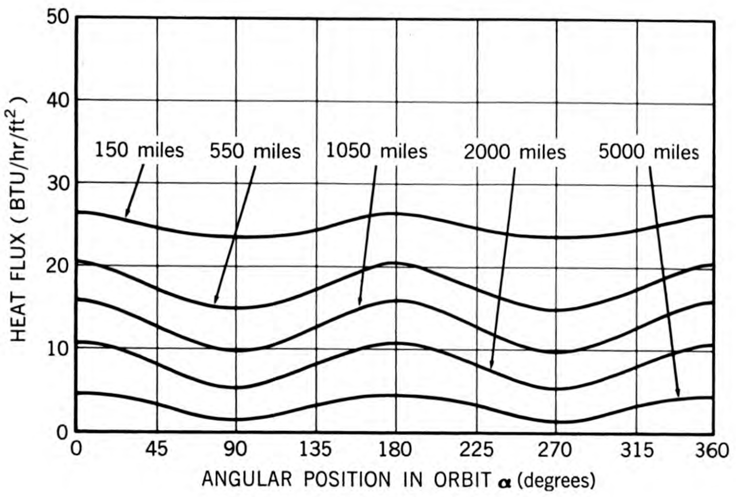

Figure 10—Earth-emitted radiation for circular orbits, δ = 90 degrees, β = 0 degrees.

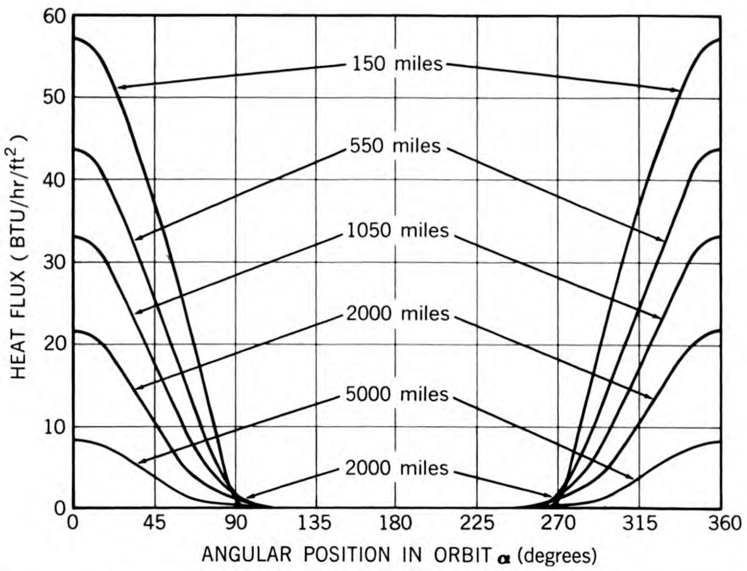

Figure 11—Albedo for minimum sunlit circular orbits, δ = 90 degrees, β = 0 degrees.

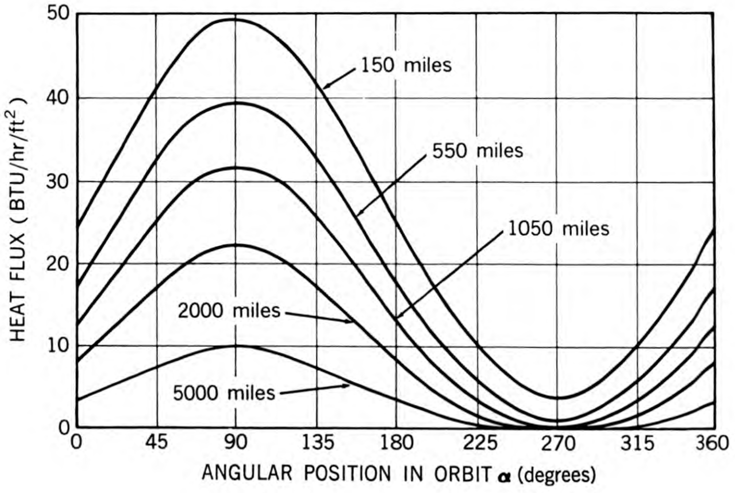

Figure 12—Earth-emitted radiation for circular orbits, δ = 90 degrees, β = 315 degrees.

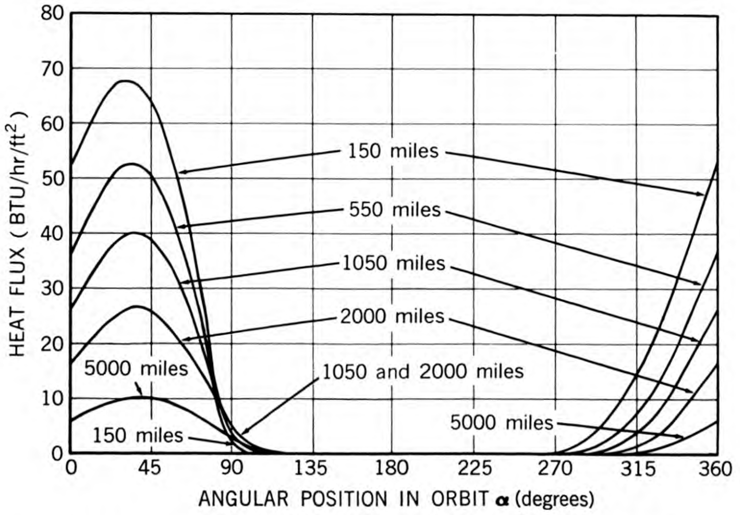

Figure 13—Albedo for minimum sunlit circular orbits, δ = 90 degrees, β = 315 degrees.

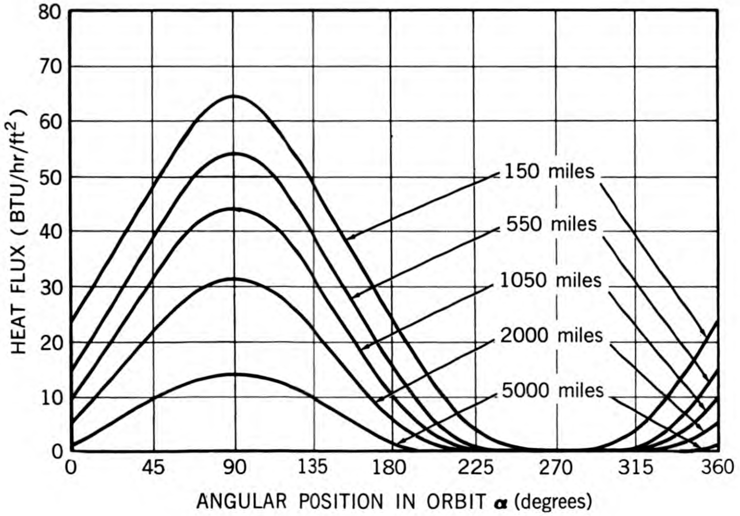

Figure 14—Earth-emitted radiation for circular orbits, δ = 90 degrees, β= 270 degrees.

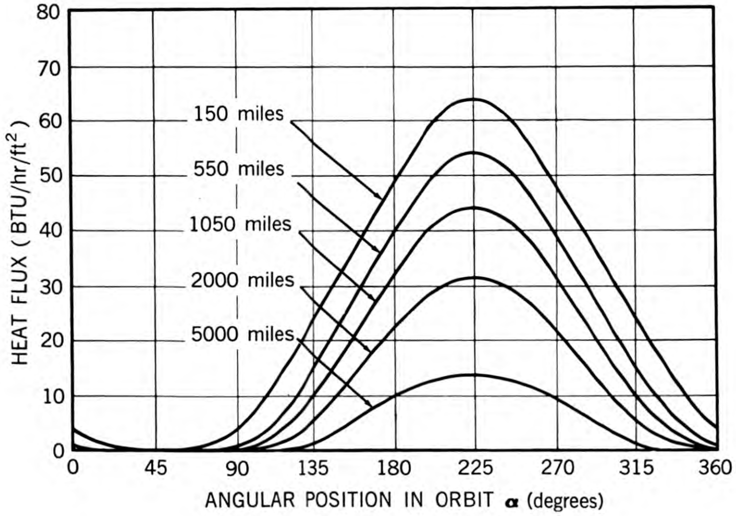

Figure 16—Earth-emitted radiation for circular orbits, δ = 45 degrees, β = 45 degrees.

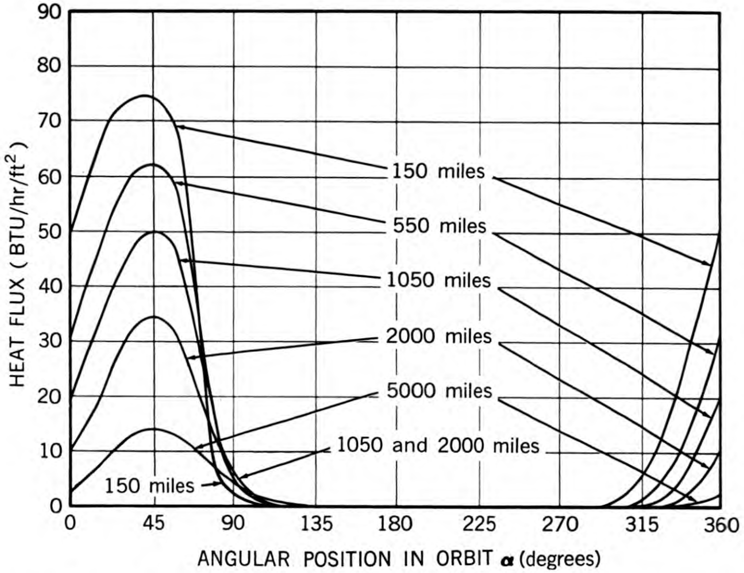

Figure 15—Albedo for minimum sunlit circular orbits, δ = 90 degrees, β = 270 degrees.

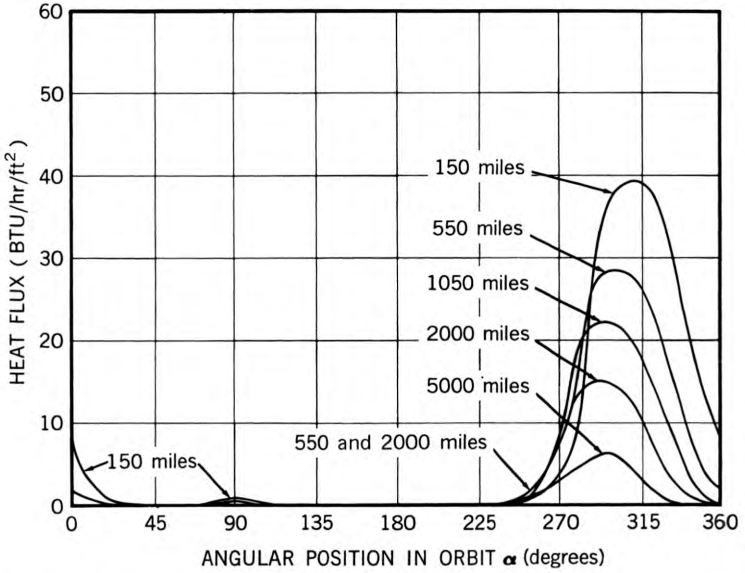

Figure 17—Albedo for minimum sunlit circular orbit, δ = 45 degrees, β = 45 degrees.

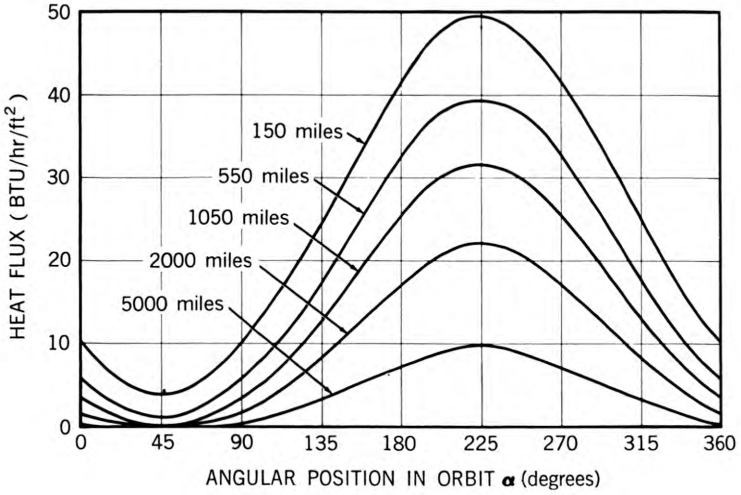

Figure 18—Earth-emitted radiation for circular orbits, δ = 45 degrees, β = 0 degrees.

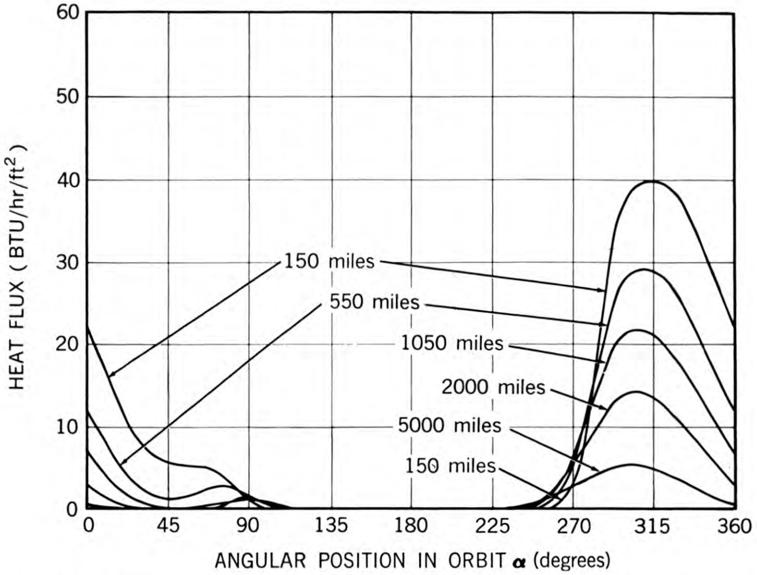

Figure 19—Albedo for minimum sunlit circular orbits, δ = 45 degrees, β = 0 degrees.

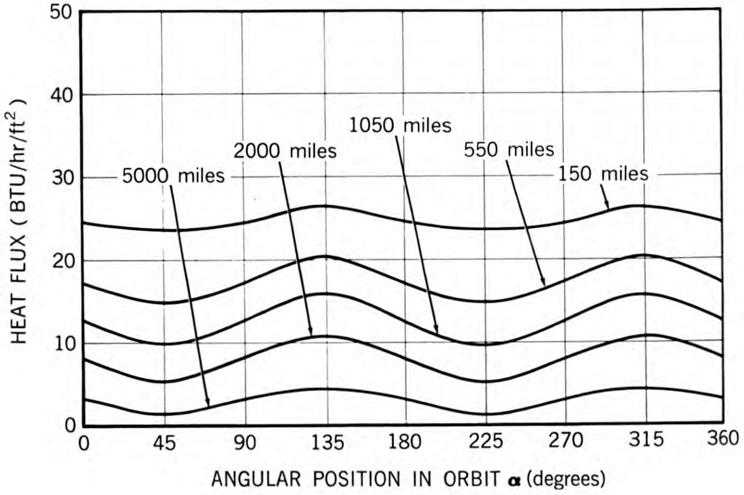

Figure 20—Earth-emitted radiation for circular orbits, δ = 45 degrees, β = 315 degrees.

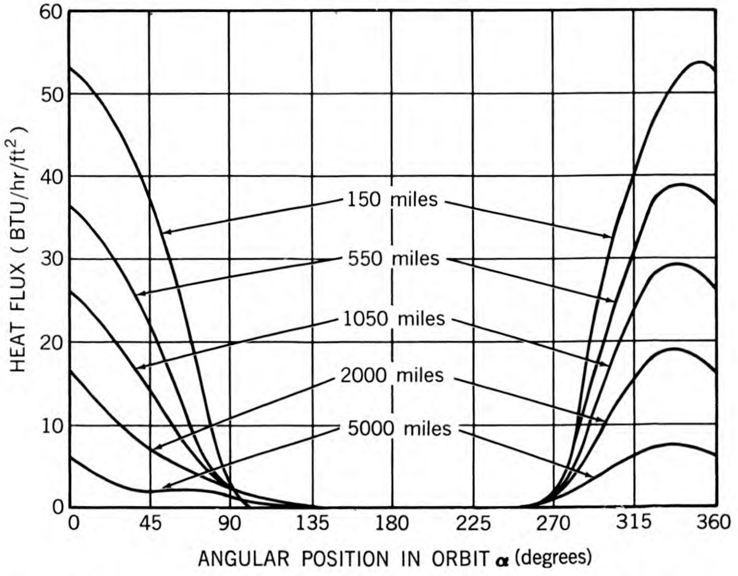

Figure 21—Albedo for minimum sunlit circular orbits, δ = 45 degrees, β = 315 degrees.

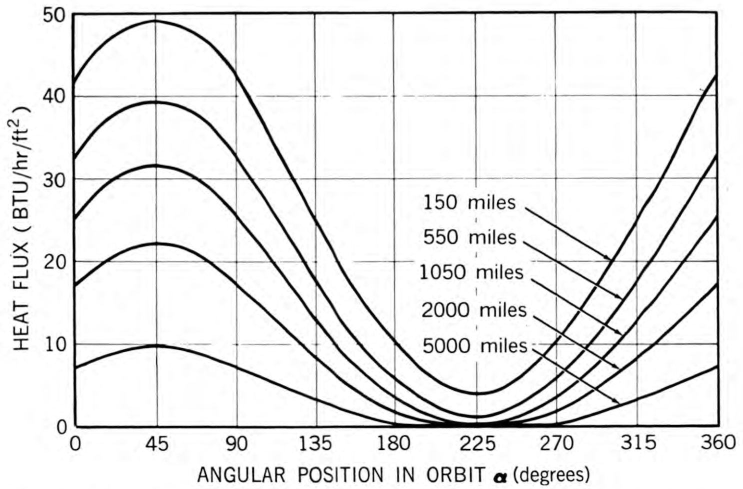

Figure 22—Earth-emitted radiation for circular orbits, $\delta = 45$ degrees, $\beta = 270$ degrees.

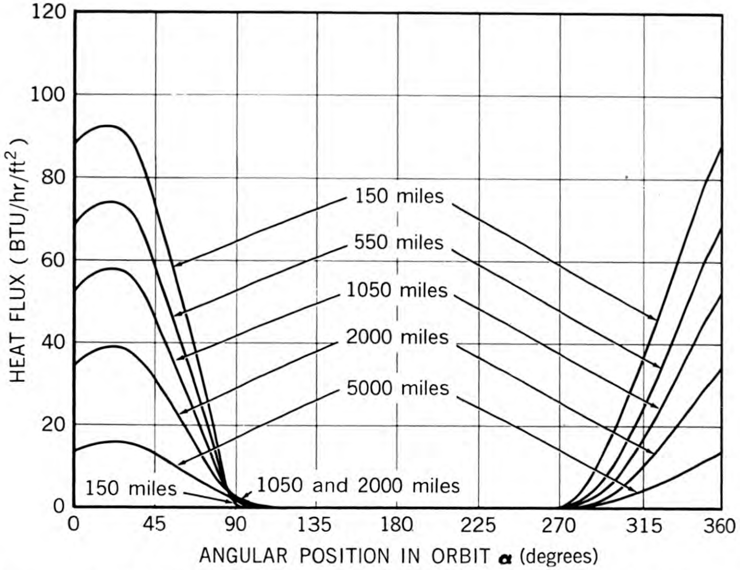

Figure 23—Albedo for minimum sunlit circular orbits, $\delta = 45$ degrees, $\beta = 270$ degrees.

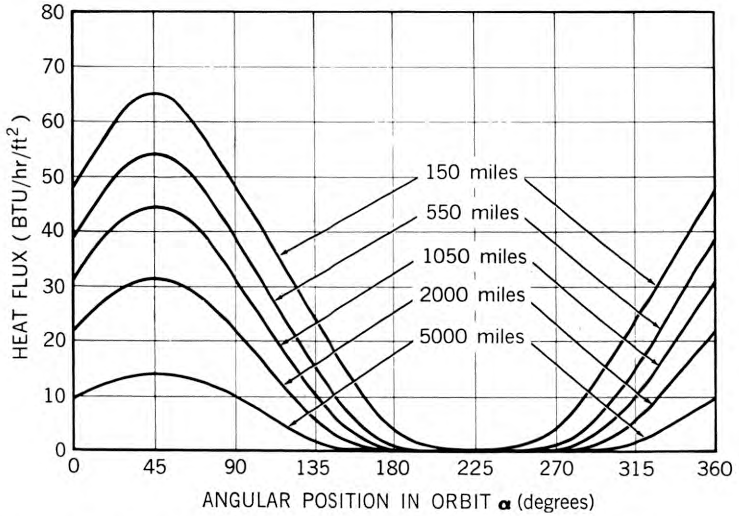

Figure 24—Earth-emitted radiation for circular orbits, $\delta = 45$ degrees, $\beta = 225$ degrees.

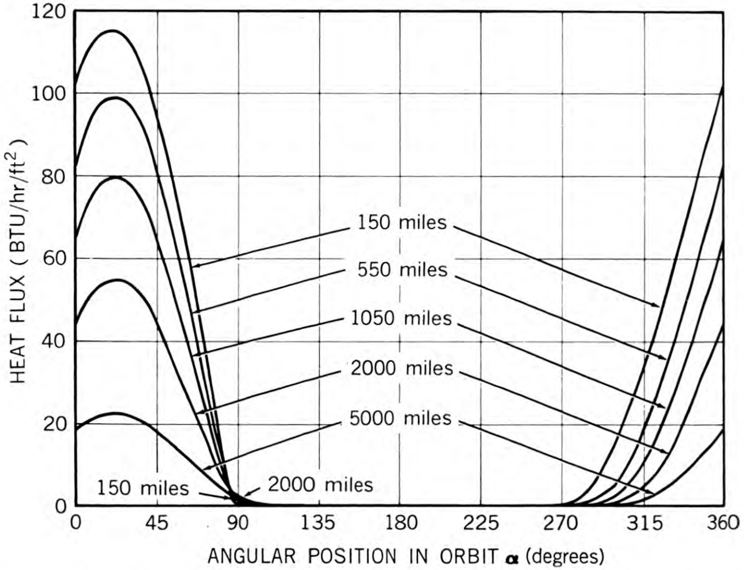

Figure 25—Albedo for minimum sunlit circular orbits, $\delta = 45$ degrees, $\beta = 225$ degrees.

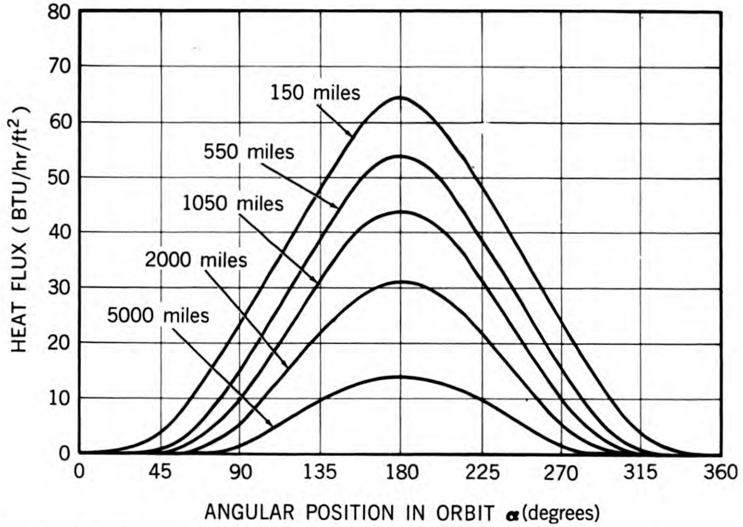

Figure 26—Earth-emitted radiation for circular orbits, δ = 0 degrees, β = 0 degrees.

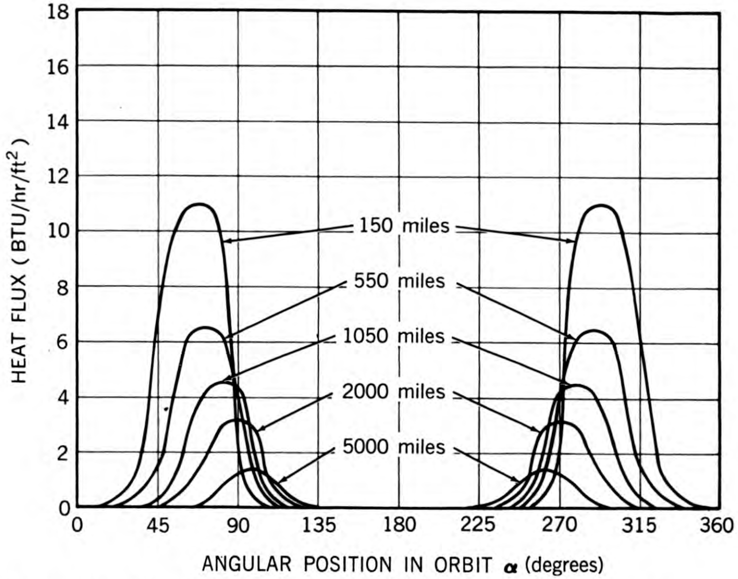

Figure 27—Albedo for minimum sunlit circular orbits, δ = 0 degrees, β = 0 degrees.

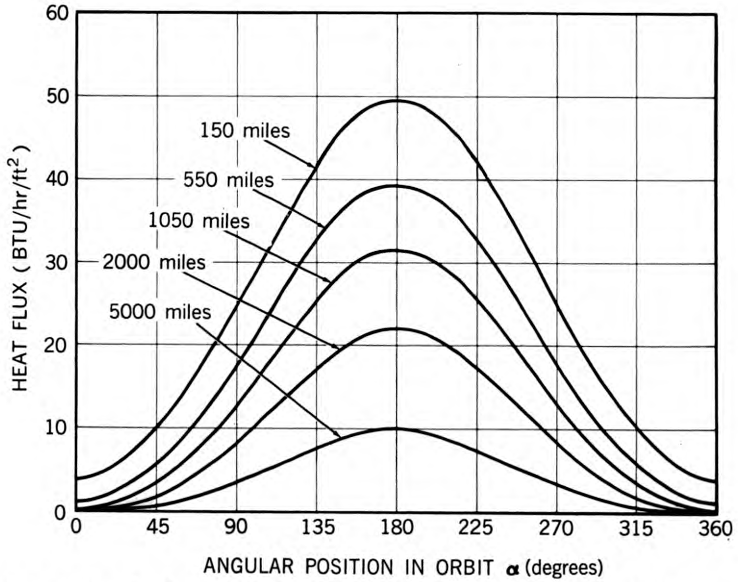

Figure 28—Earth-emitted radiation for circular orbits, δ = 0 degrees, β = 315 degrees.

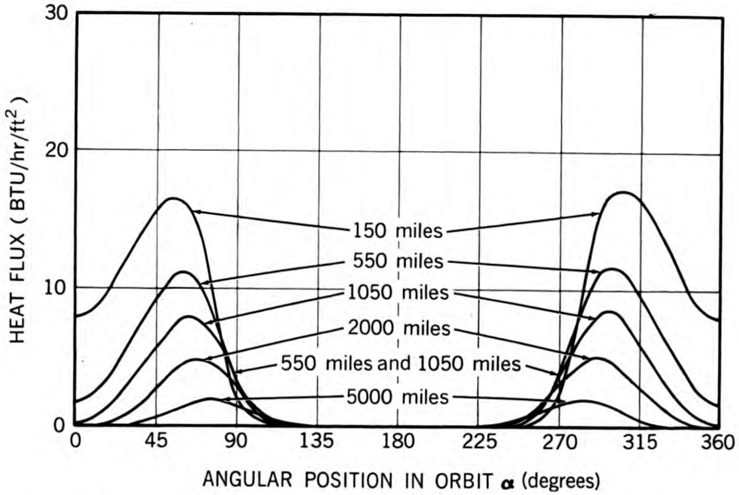

Figure 29—Albedo for minimum sunlit circular orbits, δ = 0 degrees, β = 315 degrees.

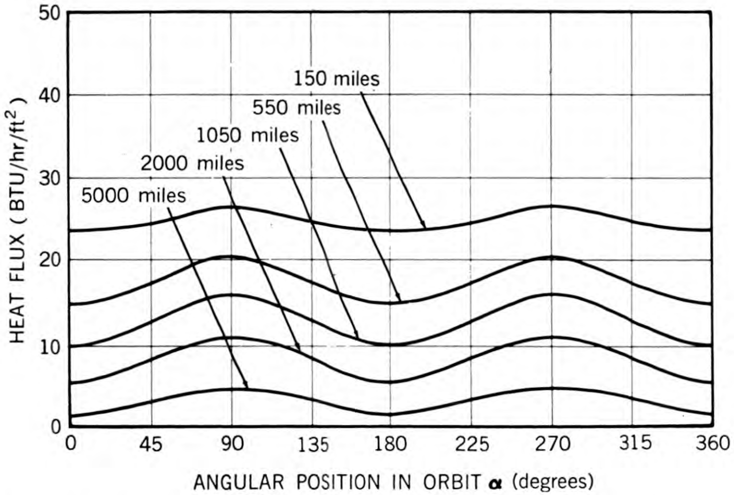

Figure 30—Earth-emitted radiation for circular orbits, δ = 0 degrees, β = 270 degrees.

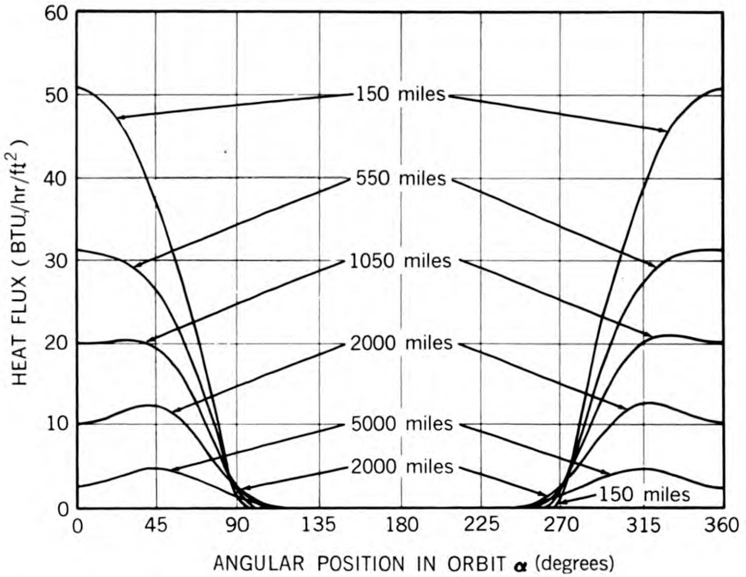

Figure 31—Albedo for minimum sunlit circular orbits, δ = 0 degrees, β = 270 degrees.

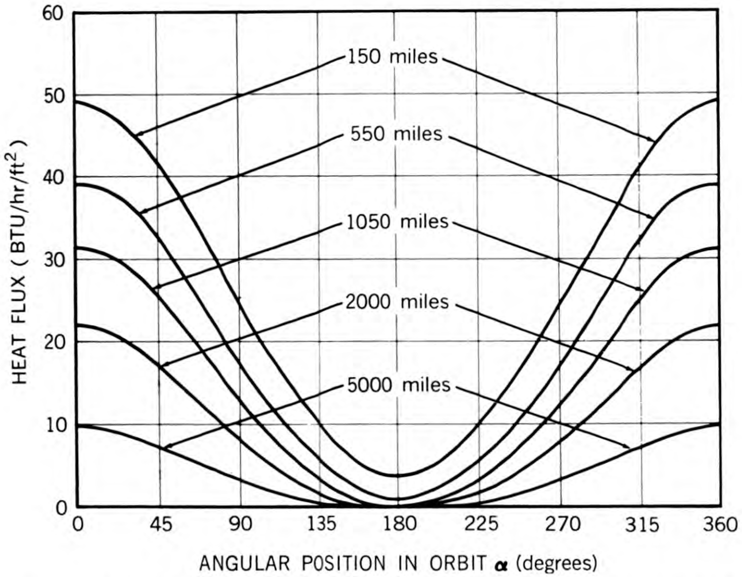

Figure 32—Earth-emitted radiation for circular orbits, δ = 0 degrees, β = 225 degrees.

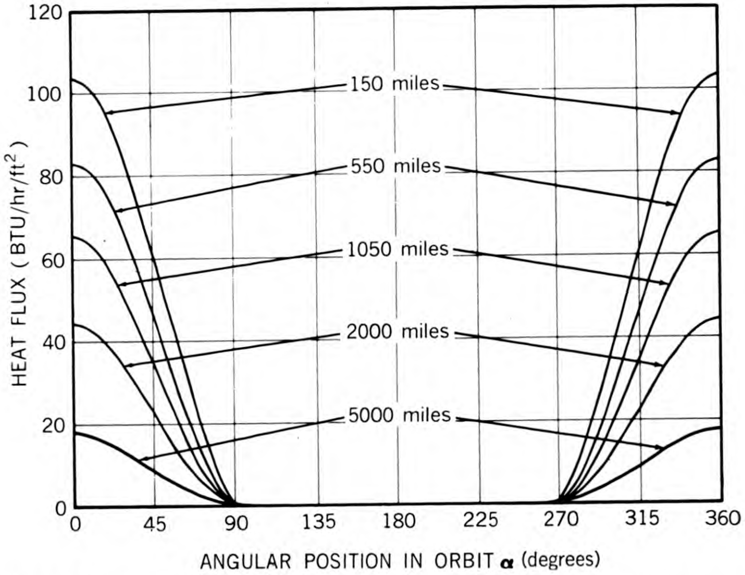

Figure 33—Albedo for minimum sunlit circular orbits, δ = 0 degrees, β = 225 degrees.

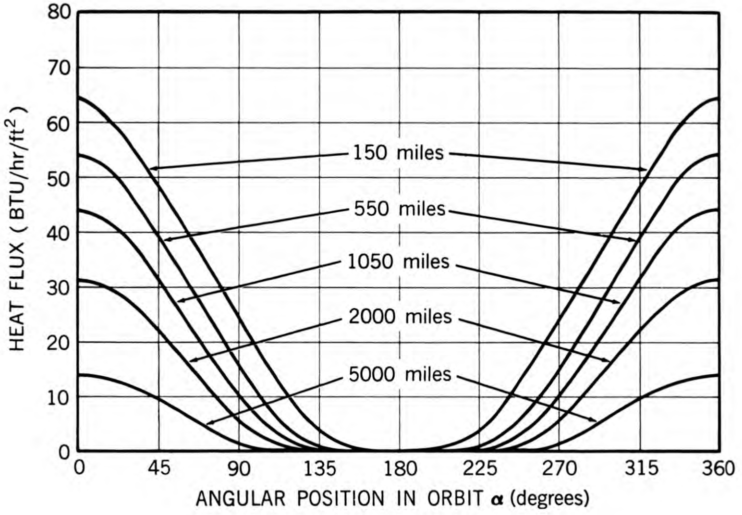

Figure 34—Earth-emitted radiation for circular orbits, δ = 0 degrees, β = 180 degrees.

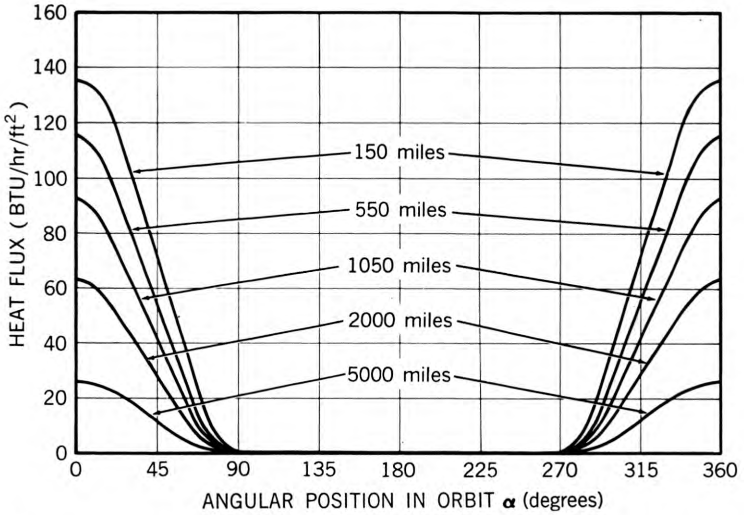

Figure 35—Albedo for minimum sunlit circular orbits, δ = 0 degrees, β = 180 degrees.

NASA TN D-2147
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(NASA TECHNICAL NOTE D-2147)

The derivation of total thermal radiation incident upon a flat plate rotating about an arbitrary axis is presented. The functional relationships between position in an elliptical earth orbit and direct solar radiation, earth-reflected solar radiation (albedo), and earth-emitted radiation (earthshine) are included. The equations have been programed for the IBM 7090 digital computer, resulting in solutions which relate the incident radiation to spin axis orientation and orbital position. Several representative orbits for a typical geometrical configuration were analyzed and are presented as examples.

I. Powers, Edward I.
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